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During the support period January through December 1989, the principal investigators ~~have studied the~~ boundary element methods, are studied for a variety of partial differential equations in optics and elasticity, including the Helmholtz equation on an exterior domain, the eigenvalue problem for the Laplacian, the elastostatic Timoshenko plate and 3-dimensional elastostatic solid mechanics. Numerical software has been accumulated and computer graphics has been successfully developed. Two papers have been published and three otheres are being prepared for publication.

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FINAL PROJECT REPORT

TO

THE AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

FOR

AFOSR Grant 88-0091

January 1, 1988 – December 1, 1988

"Computations of Optimal Controls and Designs for Distributed Systems in Optics and Elasticity"

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Table of Contents

I. Introduction

This project, entitled "Computations of Optimal Controls and Designs for Distributed systems in Optics and Elasticity", was funded by two separate grants from AFOSR (Grant #88-0091) and NSF (Grant #DMS-8718510). AFOSR has provided additional release time for Drs. G. Chen and J. Zhou, the two PI's for this project, while NSF has provided fund for the acquisition of a SUN 3/60 Workstation.

During the support period January through December 1989, the PI's have studied various questions of *controls, designs* and *computations* using the boundary element method (BEM) for distributed systems governed by the following equations in optics and elasticity:

1) the Helmholtz equation on an exterior domain;
2) the eigenvalue problem for the Laplacian;
3) the biharmonic elastostatic Timoshenko plate equation; and
4) three dimensional solid mechanics equations.

Numerical software has been accumulated and computer *graphics* has been successfully developed. So far, two papers have been published. Three others are being prepared for publication. See §II and §III below.

This research project has been consolidated with our AFOSR Grant 87-0334 and will be carried on further in that grant.

§II. List of Publications

The following research papers have been published under the auspices of this grant:

[1] **Computing optimal boundary controls of a plate by the boundary element method** (authors: G. Chen and J. Zhou)

Status: presented at the 26th IEEE Control and Decision Conference, Los Angeles, December 1987, and appeared in the Proceedings, pp. 992-996.

To our knowledge, this is the *first* successful computation of the biharmonic Timoshenko plate equation by the boundary element method. Optimal boundary controls in terms of displacement and angle are also computed.

[2] **Minimizing the reflection of waves by surface impedance using boundary elements and global optimization** (authors: G. Chen, T.J. Bridges and J. Zhou)
Status: published in Wave Motion 10 (1988), pp. 239-255. The material was also presented at an unscheduled talk in the Session on Control of Distributed Parameter Systems in the 26th IEEE-CDC in Los Angeles in December 1987.

In this paper, we used the boundary element method and global optimization techniques to study the "stealth problem", i.e., the minimization of reflected radar or sonar waves. All of the computer codes, including the Helmholtz equation solver on an exterior domain and the global optimization algorithms, were developed by us. Numerical results have been benchmarked and have conformed to the physical circumstances.

§III. Papers in Preparation

The following papers are presently being written for submission to scientific journals. Preprints will be sent to the Program Manager at AFOSR once they become available.

[1] **Boundary integral equations and boundary element computations of the Timoshenko plate equation with applications to boundary shape control**

This is a long technical paper where we have formulated the boundary integral equations for the elastostatic biharmonic plate equation subject to four types of combinations of displacement, angle, bending moment and shear boundary conditions. Boundary element computations have been successfully carried out and tested. The methods are then applied to boundary shape controls and numerical modelling of the control system.

We expect this work to be complete by early May 1989.

[2] **The boundary element method for shape control of distributed parameter systems**

Drs. G. Chen and J. Zhou have been invited by Professor John L. Junkins to write a survey chapter for a new volume entitled "*Mechanics and Control of Space Structure*" in the AIAA "Progress in Astronautics and Aeronautics Series", for which Professor Junkins is the editor. This volume will summarize the state of arts of active control of space structure.

The due date of the first draft of the manuscript is June 1, 1989.

[3] The eigenvalue problem and eigenshapes for the Laplacian

Traditionally, the computation of eigenvalues and eigenfunctions of the Laplacian on nonrectangular domains is done by the finite element method, which involves a large amount of coding work, numerical quadrature and computer storage. We have developed boundary element methods and algorithms for studying such problems, which are accurate and fast. Computer graphics for eigenshapes has also been successfully produced, which displays remarkable accuracy in comparison with the benchmarks.

This work is expected to be complete by October 1989.

[4] The boundary element method and shape control for 3-D solid mechanics

The SUN 3/60 Workstation acquired with NSF Grant DMS 87-18510 has been installed in our office since May 1988. We are now able to run the machine with UNIX and have become familiar with SUN's 3-D color graphics package for our visualization purpose. For 3-D solid mechanics, the governing equations are

$$\sum_{j=1}^{3} \frac{\partial}{\partial x_j} \sigma_{ij}(x) = f_i(x), \qquad x \in \Omega \subseteq \mathbf{R}^3, i = 1,2,3$$

where

$$\begin{aligned} \sigma_{ij}(x) &= \text{ the linear stress tensor,} \\ &= \lambda[\nabla \cdot \overline{u}(x)]\delta_{ij} + 2\mu\varepsilon_{ij}(x), \\ \varepsilon_{ij}(x) &= \text{ the linear strain tensor,} \\ \overline{u}(x) &= \text{ the displacement vector,} \\ &\lambda, \mu \text{ are Lame's constants,} \\ &f_i(x),\ i = 1,2,3, \text{ are components of body force.} \end{aligned}$$

A boundary element package has been developed. To illustrate our progress and achievement, we include the graphics from a simple numerical example, using

$$\vec{f} = (f_1, f_2, f_3) \equiv 0$$

$$\lambda = \frac{10}{3}, \mu = 1$$

$$\Omega = \text{ the unit ball}$$

$$\vec{\tau} = \text{ the boundary traction force}$$

$$= (\tau_1, \tau_2, \tau_3)$$

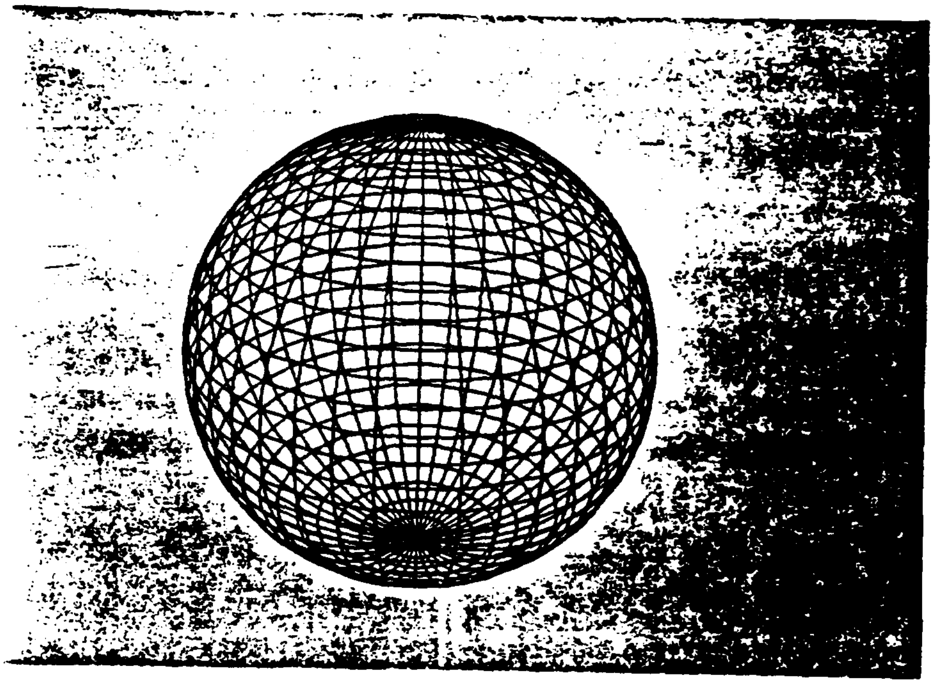
$$\tau_1 = \tau_2 \equiv 0 \qquad \text{on} \quad \partial\Omega$$

$$(*) \qquad \tau_3 = \begin{cases} -(1 - \sin\phi), & 0 \le \phi \le \pi/2 \\ 1 - \sin\phi, & \frac{\pi}{2} \le \phi \le \pi \end{cases}, \quad \phi \text{ is the latitudinal angle.}$$

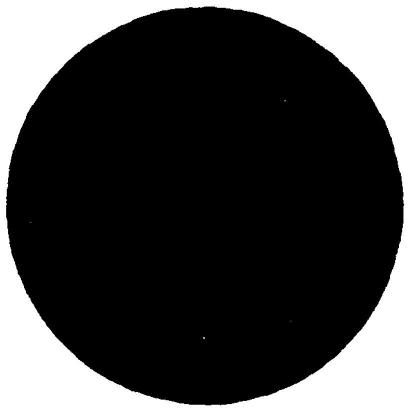
Therefore the ball is subject to a vertical compression force. In Fig. 1 (a) and (b), we see the unit ball, whose surface is discretized into $30 \times 40 = 1,200$ piecewise constant boundary element. In Fig. 2 (a) and (b), we see that the ball has been compressed into an ellipsoidal shape. In Fig. 3 (a) and (b), we see that the ball has been further compressed into an "English muffin" if the vertical compression force (*) is tripled.

This research of boundary element computations and applications to solid mechanics control was presented by G. Chen, L. Ji and J. Zhou at an Invited Session on Inverse Problems at the 27th IEEE-CDC, Dec. 7-9, 1988, in Austin, Texas.

A research paper is being prepared for publication.

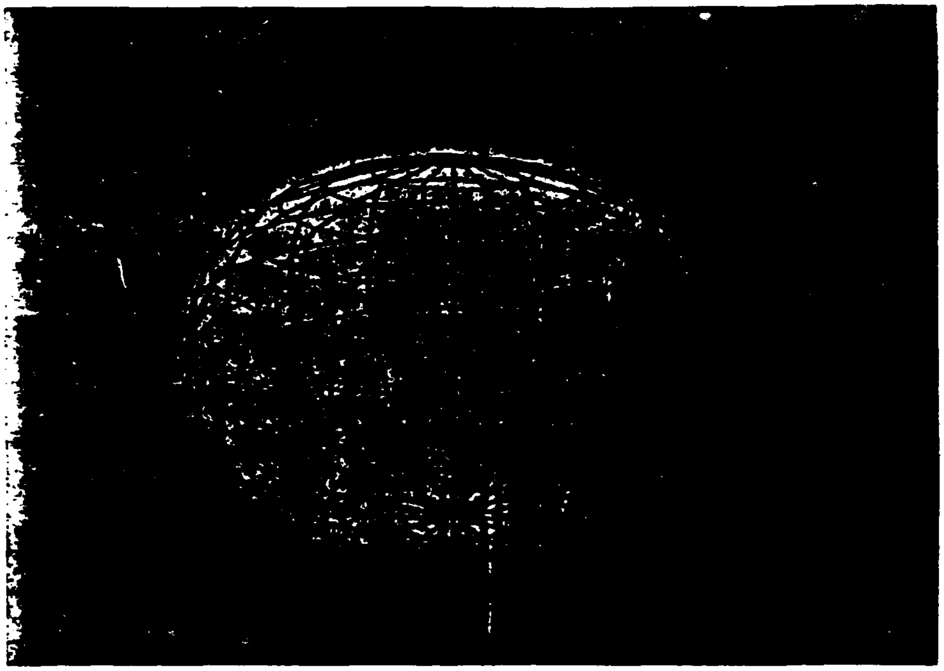


(a)

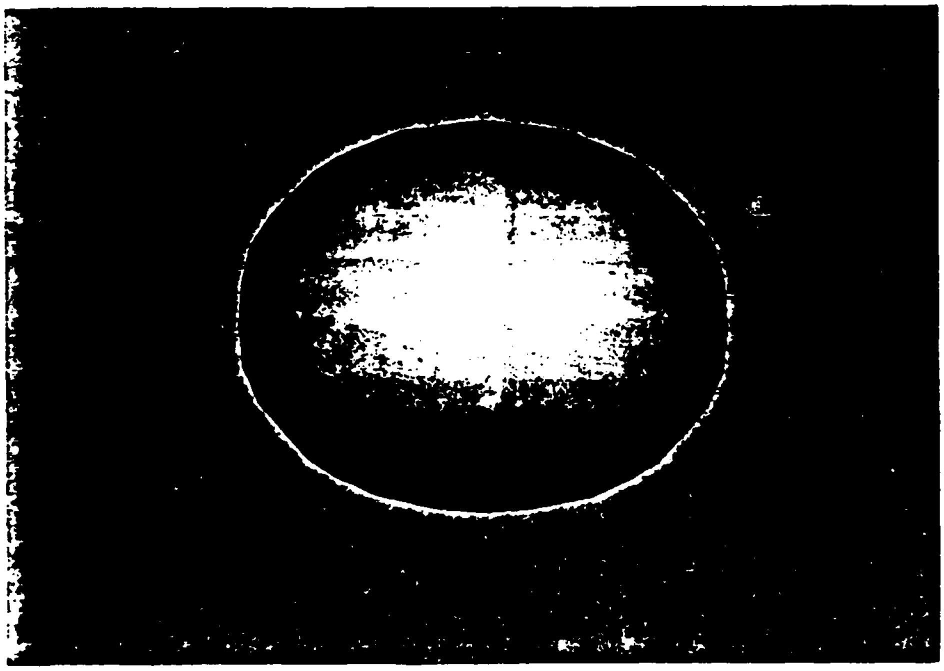


(b)

Fig. 1 In (a) and (b), the surface of the unit sphere is discretized into 30 × 40 piecewise constant boundary elements



(a)

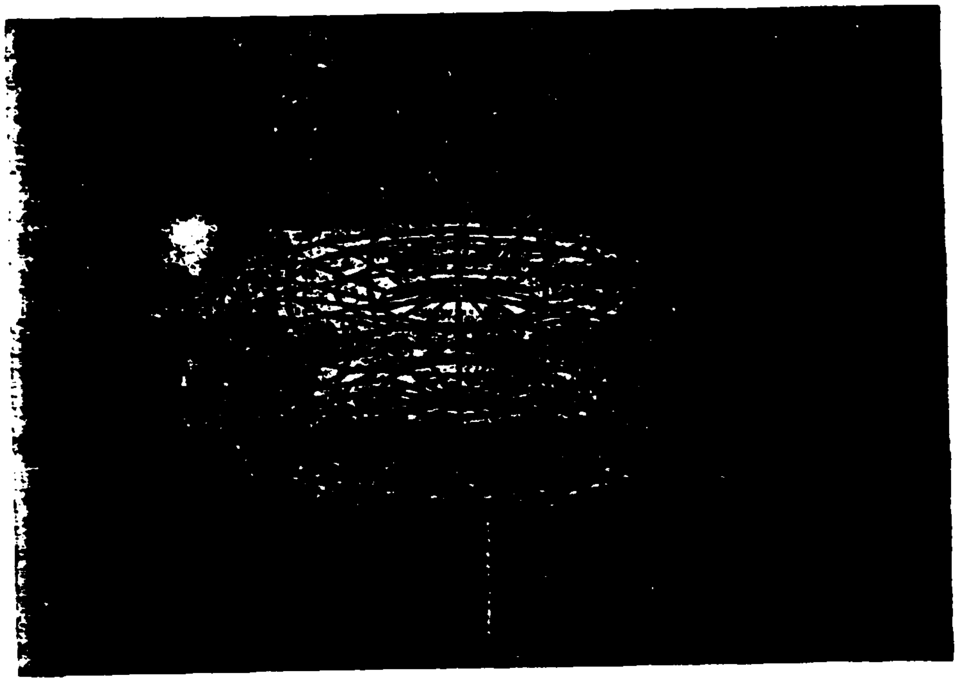


(b)

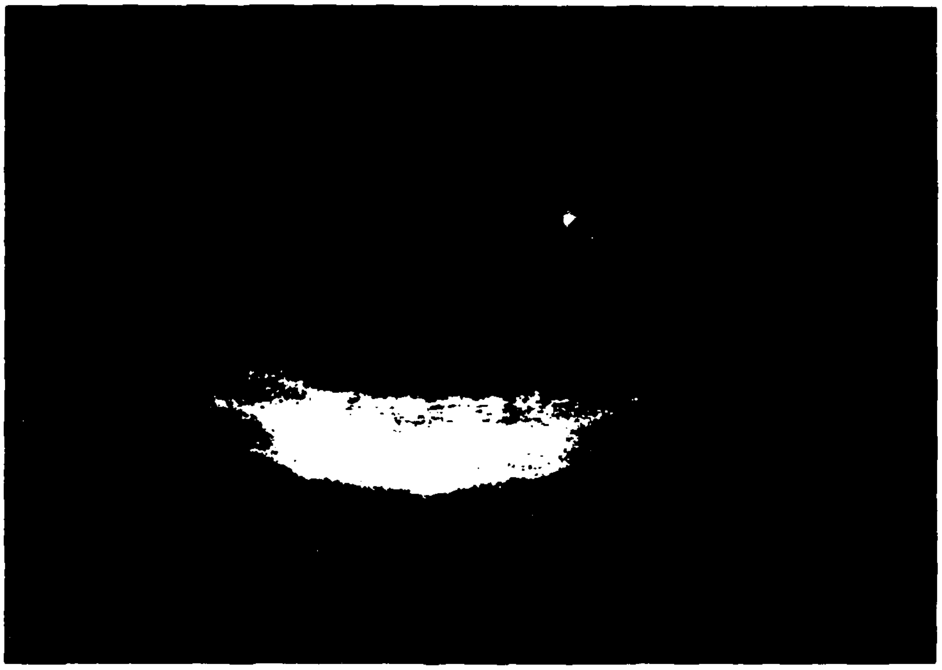
Fig. 2 The deformed unit ball takes an ellipsoidal shape, where the surface is subject to a compressive traction force

$$\vec{t}_1(\theta,\phi) = \begin{cases} -(1-\sin\phi), & 0<\phi<\pi/2 \\ 1-\sin\phi, & \pi/2<\phi<\pi \end{cases}, \quad \theta=\text{longitutinal angle},\ \phi=\text{latitudinal angle}$$

15 x 15 = [illegible]



(a)



(b)

Fig. 3 The deformed unit ball takes the shape of a "muffin", where the surface is subject to a compressive traction force three times of that in Fig. 2. Here the extent of deformation actually has exceeded the linear elasticity limit. The purpose of this figure is to illustrate the dramatic effects.

§IV. Personnel

Dr. G. Chen, the PI, supervises the overall conduct of the grant. Dr. J. Zhou is the co-PI. The following people are collaborators associated with the research effort:

T.J. Bridges - Assistant Professor, Department of Mathematics, Worcester Polytechnic Institute, Worcester, MA 01609

L. Ji – Ph.D. student, Department of Mathematics, Texas A&M University, College Station, TX 77843

§V. Activities

Drs. Chen and Zhou have given the following talks on their collaborated work:

[1] The 26th IEEE-CDC, Los Angeles, CA, Dec. 1987.
Subject matter: Computing optimal boundary controls of a plate by the boundary element method.
Individual involved: Prof. J.E. Lagnese, Georgetown University and Prof. M.C. Delfour, Univeristy of Montreal, Canada.

[2] Workshop on Numerical and Experimental Aspects of Distributed Parameter Control, Center for Mathematical Sciences, University of Wisconsin, Madison, WI, May 1988.
Subject matter: The boundary element method for computing boundary controls in optics and elasticity.
G. Chen is a major invited speaker in the Workshop, wherein J. Zhou gave a demonstration of computer graphics.
Individual involved: Prof. D.L. Russell, University of Wisconsin.

[3] Conference in honor of the 65th birthday of Regent Professor Lawrence Markus, Dept. of Math., University of Minnesota, Minneapolis, April 1988.
Subject Matter: Boundary control of a thin elastostatic plate by the boundary element method.
G. Chen was an invited keynote speaker for the conference.
Individual involved: Prof. E.B. Lee, Electrical Engineering Dept., University of Minnesota.

[4] The 27th IEEE-CDC, Austin, Texas, Dec. 1988.

Subject matter: The boundary element method for shape control in 3-D solid **mechancs.**

Individual involved: Prof. G. Crosta, University of Milan, Italy.